CBSE Board
Class XI Mathematics
Sample Paper – 8

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Find $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$.

2. Write the negation of the statement: The number 2 is greater than 7.

3. Find the value of $\frac{-1}{i}$.

OR

Find modulus of 1 – 3i.

4. If variance of a distribution is 4 then find standard deviation of the distribution.

SECTION – B

5. If A = {a, b, c, d}, B = {f, b, d, g} and n(A ∪ B) = total number of elements = 8 then find n(A' ∪ B').

6. Let f : R → R be a function given by $f(x) = x^2 + 1$, find : $f^{-1}(-5)$.

OR

If $f(x) = \frac{x-1}{x+1}$ then show that f(1/x) = -f(x).

7. Find the radian measures corresponding to the $5^\circ 37' 30''$.

OR

Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

8. If A = {1, 4}, B = {2, 3, 6} and C = {2, 3, 7} verify that $A \times (B - C) = A \times B - A \times C$

9. Solve $\sin^2 x + \sin x - 2 = 0$ where $0^\circ < \theta < 360^\circ$

OR

Prove that $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=2\sec\theta$

10. Given below are two statements :

p: 25 is a multiple of 5

q: 25 is a multiple of 8

Write the compound statement connecting these two statements with "OR" and check its validity.

11. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

12. In what ratio the line joining (-1, 1) and (5, 7) is divided by the line $x + y = 4$.

SECTION – C

13. Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

14. Which of the following relations are functions?

1. A is the capital of b where $b \in B$ and B is the set of all countries, $a \in A$ and A is the set of capital cities of countries.
2. $y < x + 3$
3. y is a Maths pupil of x, where x represents any Maths teacher in a school.
4. $y = 3x + 2$

15. Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$ i. e. $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$ Express R and R^{-1} as sets of ordered pairs. Also determine domains of R and R^{-1} and ranges of R and R^{-1}.

16. If $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A. P. then find the value of x.

17. If 1, ω, ω^2 are the cube roots of unity, prove that

$(1 + \omega)^3 - (1 + \omega^2)^3 = 0$ and $(x - y)(x\omega - y)(x\omega^2 - y) = x^3 - y^3$

18. An integer is chosen at random from the first two hundred positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

19. Evaluate $0.23\overline{45}$

20. Determine the number of natural numbers smaller than 10^4, in the decimal notation of which all the digits are distinct.

OR

Find the sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time.

21. Find the equation of the locus of all points such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2.

OR

Find the lengths of the transverse and conjugate axes, co-ordinates of the foci, vertices and eccentricity for the hyperbola $9x^2 - 16y^2 = 144$

22. Evaluate the derivative of the following function at indicated points :

$\frac{1-\sin x}{1+\cos x}$ at $x = \frac{\pi}{2}$

OR

If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ show that $\frac{dy}{dx} = y$

23. Find the vertex, focus and directrix of the parabola $4y^2 + 12x - 12y + 39 = 0$.

SECTION – D

24. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = 1/16$

OR

Prove that $\cos A \cos(60° - A) \cos(60° + A) = \frac{1}{4}\cos 3A$

25. For a group of 200 candidates the mean and S. D. were found to be 40 and 15 respectively. Later on it was found that the score 43 was misread as 34. Find the correct mean and correct S. D.

26. If $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$ prove that $\sin\theta = \frac{3\sin\alpha + \sin^3\alpha}{1 + 3\sin^2\alpha}$

27. Solve the following system of inequalities graphically:

$x + 2y \le 10; x + y \ge 1; x - y \le 0; x \ge 0; y \ge 0$

OR

For the purpose of an experiment an acid solution between 4% and 6% is required. 640 liters of 8% acid solution and a 2% acid solution are available in a laboratory. How many liters of the 2% solution needs to be added to the 8% solution?

28. Show by mathematical induction that the sum to n terms of the series

$1^2 + 2\times 2^2 + 3^2 + 2\times 4^2 + 5^2 + 2\times 6^2 + ...$is

$$S_n = \begin{cases} \frac{n(n+1)^2}{2}, & \text{when n is even} \\ \frac{n^2(n+1)}{2}, & \text{when n is odd} \end{cases}$$

29. A student wants to buy a computer for Rs. 12,000. He has saved up to Rs. 6000 which he pays as cash. He is to pay the balance in annual installments of Rs. 500 plus an interest of 12% on the unpaid amount. How much will the computer cost him?

OR

Find the value of $\frac{1\times 2^2 + 2\times 3^2 + 3\times 4^2 + ...\text{uptill the nth term}}{1^2\times 2 + 2^2\times 3 + 3^2\times 4 + ...\text{uptill the nth term}}$.

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SECTION – A

1.

$$\lim_{x\to\frac{1}{2}}\frac{4x^2-1}{2x-1}$$
$$=\lim_{x\to\frac{1}{2}}\frac{(2x+1)(2x-1)}{2x-1}$$
$$=\lim_{x\to\frac{1}{2}} 2x+1 \qquad 2x-1\neq 0$$
$$=2$$

2. The number 2 is not greater than 7.
OR
The number 2 is less than or equal to 7.
OR
It is false that the number 2 is greater than 7.

3.

$$\frac{-1}{i}=\frac{-1}{i}\times\frac{i}{i}=\frac{-i}{i^2}=\frac{-i}{-1}=i$$

OR

z = 1 – 3i
Comparing with a + bi we get a = 1 and b = -3

$$|z|=\sqrt{1^2+(-3)^2}=\sqrt{10}$$

4. Variance = 4
Standard deviation = √Variance
Standard deviation = √4
Standard deviation = 2

SECTION – B

5. According to the question,

$A \cap B = \{b, d\}$

$n(A \cap B) = 2$

$n(A' \cup B') = n(A \cap B)' = n(A \cup B) - n(A \cap B) = 8 - 2 = 6$

6. $f^{-1}(-5) = x$. Then,

$f(x) = -5$

$x^2 + 1 = -5$

$x^2 = -6$

This equation is not solvable in R. Therefore, there is no pre-image of -5. Hence, $f^{-1}(-5) = \phi$

OR

$$f(x) = \frac{x-1}{x+1}$$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1}$$

$$f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$$

$$f\left(\frac{1}{x}\right) = \frac{-(x-1)}{x+1}$$

$$f\left(\frac{1}{x}\right) = -f(x)$$

7. $30'' = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$

$$37'\,30'' = \left(37\frac{1}{2}\right)' = \left(\frac{75}{2}\right)' = \left(\frac{75}{2} \times \frac{1}{60}\right)^\circ = \left(\frac{5}{8}\right)^\circ$$

OR

Let s be the length of the arc subtending an angle θ^c at the centre of a circle of radius r.

$s = r\theta$ here $r = 5$ cm and $\theta = 15^\circ$

$$s = r\theta = 5 \times 15 \times \frac{\pi}{180} = \frac{5\pi}{12}$$

8. B – C = {2, 3, 6} – {2, 3, 7} = {6}

A × (B – C) = {1, 4} × {6} = {(1, 6), (4, 6)}.......(i)

A × B = {1, 4} × {2, 3, 6} = {(1, 2), (1, 3), (1, 6), (4, 2), (4, 3), (4, 6)}

A × C = {1, 4} × {2, 3, 7} = {(1, 2), (1, 3), (1, 7), (4, 2), (4, 3), (4, 7)}

A × B – A × C = {(1, 6), (4, 6)}.....(ii)

From (i) and (ii)

A × (B – C) = A × B – A × C

9. $\sin^2 x + \sin x - 2 = 0$

$(\sin x + 2)(\sin x - 1) = 0$

$\sin x + 2 = 0$ or $\sin x - 1 = 0$

$\sin x = -2$ or $\sin x = 1$

sin x cannot be -2 hence, sin x = 1.

$x = 90^\circ$

OR

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$$

$$=\frac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{\left(1+\tan\frac{\theta}{2}\right)\left(1-\tan\frac{\theta}{2}\right)}$$

$$=\frac{2\left(1+\tan^2\frac{\theta}{2}\right)}{1-\tan^2\frac{\theta}{2}}$$

$$=\frac{2}{\cos\theta}$$

$$=2\sec\theta$$

10. The compound statement is "25 is a multiple of 5 or 8".

Let us assume that the statement q is false i. e. 25 is not a multiple of 8. Clearly p is true.

Thus, if we assume that q is false then p is true. Hence, compound statement is true.

11. The values of x for which f(x) and g(x) are equal given by

$f(x) = g(x)$

$2x^2 - 1 = 1 - 3x$

$2x^2 + 3x - 2 = 0$

$(x + 2)(2x - 1) = 0$

$x = -2, \frac{1}{2}$

Thus, f(x) and g(x) are equal on the set $\{-2, \frac{1}{2}\}$.

12. Suppose the line $x + y = 4$ divides the join of A(-1, 1) and B(5, 7) in the ratio k : 1.

The coordinate of the point of division are $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$

It lies on $x + y = 4$.

$$\frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$$

$5k - 1 + 7k + 1 = 4k + 4$

$12k - 4k = 4$

$8k = 4$

$k = \frac{1}{2}$

SECTION – C

13. $$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{\cos 4\theta} - 1}$$

$$= \frac{(1 - \cos 8\theta)\cos 4\theta}{\cos 8\theta(1 - \cos 4\theta)}$$

$$= \frac{2\sin^2 4\theta \cos 4\theta}{\cos 8\theta 2\sin^2 2\theta}$$

$$= \frac{2\sin 4\theta \cos 4\theta}{\cos 8\theta} \times \frac{\sin 4\theta}{2\sin^2 2\theta}$$

$$= \frac{2\sin 4\theta \cos 4\theta}{\cos 8\theta} \times \frac{2\sin 2\theta \cos 2\theta}{2\sin^2 2\theta}$$

$$= \frac{\sin 8\theta}{\cos 8\theta} \times \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \tan 8\theta \times \cot 2\theta$$

$$= \frac{\tan 8\theta}{\tan 2\theta}$$

14.

1. It is a function. Because normally each country has exactly one capital city.
2. It is not a function.
   For each replacement of x there are many replacements of y which makes the sentence true.
3. It is not a function.
   Each Maths teacher ordinarily has a number of pupils.
4. It is a function.
   Each replacement of x gives one and only one replacement of y which makes the sentence true.

15. $(x, y) \in R \Leftrightarrow x + 2y = 10 \Leftrightarrow y = \frac{10-x}{2}$, $x, y \in A$ where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$x = 1 \Rightarrow y = 9/2 \notin A$

This shows that 1 is not related to any element in A. Similarly we can observe that 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation.

Further we find that

For $x = 2$, $y = 4 \in A$ therefore, $(2, 4) \in R$

For $x = 4$, $y = 3 \in A$ therefore, $(4, 3) \in R$

For $x = 6$, $y = 2 \in A$ therefore, $(6, 2) \in R$

For $x = 8$, $y = 1 \in A$ therefore, $(8, 1) \in R$

Thus, $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\} \Rightarrow R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$

Clearly, $Dom(R) = \{2, 4, 6, 8\} = Range\ R^{-1}$ and $Range\ (R) = \{4, 3, 2, 1\} = Dom\ (R^{-1})$

16. $\log_{10} 2$, $\log_{10} (2^x - 1)$ and $\log_{10} (2^x + 3)$ are in A. P.

$2\log_{10} (2^x - 1) = \log_{10} 2 + \log_{10} (2^x + 3)$

$\log_{10} (2^x - 1)^2 = \log_{10} [2(2^x + 3)]$

$(2^x - 1)^2 = 2(2^x + 3)$

$(y - 1)^2 = 2(y + 3)$ $\quad \because y = 2^x$

$y^2 - 2y + 1 = 2y + 6$

$y^2 - 4y - 5 = 0$

$(y - 5)(y + 1) = 0$

$Y = 5, -1$

When $y = 5$, then $2^x = 5 \therefore x = \log_2 5$

When $y = -1$ then $2^x = -1$ which is impossible.

Hence, $x = \log_2 5$

17. $(1+\omega)^3 - (1+\omega^2)^3 = (-\omega^2)^3 - (-\omega)^3$

$= -\omega^6 + \omega^3$

$= -(\omega^3)^2 + \omega^3$

$= -1 + 1$

$= 0$

$\therefore (1+\omega)^3 - (1+\omega^2)^3 = 0$

$(x-y)(x\omega - y)(x\omega^2 - y) = (x-y)(x^2\omega^3 - xy\omega - xy\omega^2 + y^2)$

$= (x-y)[x^2 - xy(\omega+\omega^2) + y^2]$

$= (x-y)[x^2 - xy(-1) + y^2]$

$= (x-y)(x^2 + xy + y^2)$

$= x^3 - y^3$

$\therefore (x-y)(x\omega - y)(x\omega^2 - y) = x^3 - y^3$

18. Total number of exhaustive , mutually exclusive and equally likely cases, i. e. n(S) = 200
Integers from 1 to 200 divisible by 6 are 6, 12, ...198. Their number = 198/6 = 33
Integers from 1 to 200 divisible by 8 are 8, 16,....,200. Their number = 200/8 = 25
Integers from 1 to 200 divisible by both 6 and 8 are 24, 48,...192. Their number = 192/24 = 8
These 8 numbers have been counted twice, in multiple of 6 and also in multiple of 8
Favourable number of cases = 33 + 25 – 8 = 50
Required probability = 50/200 = ¼

19. $0.23\overline{45} = 0.2345454545...\text{to } \infty$

$= 0.23 + 0.0045 + 0.000045 + 0.00000045...\text{to } \infty$

$= \frac{23}{100} + \frac{45}{10000} + \frac{45}{1000000} + \frac{45}{100000000}...\text{to } \infty$

$= \frac{23}{100} + \frac{45}{10000}\left(1 + \frac{1}{100} + \frac{1}{1000}...\text{to } \infty\right)$

$= \frac{23}{100} + \frac{45}{10000}\left(\frac{1}{1 - \frac{1}{100}}\right)$

$= \frac{23}{100} + \frac{45}{10000}\left(\frac{100}{99}\right)$

$= \frac{23}{100} + \frac{45}{9900}$

$= \frac{23}{100} + \frac{5}{1100}$

$= \frac{258}{1100} = \frac{129}{550}$

20. The required number consist of 1-digit, 2-digits, 3-digits and 4-digits.

Total number of one-digit natural number = 9

Total number of two-digit natural numbers = ${}^{10}P_2 - {}^{9}P_1$

Total number of three-digit natural numbers = ${}^{10}P_3 - {}^{9}P_2$

Total number of four-digit natural numbers = ${}^{10}P_4 - {}^{9}P_3$

Hence, the required number of natural numbers

$= 9 + {}^{10}P_2 - {}^{9}P_1 + {}^{10}P_3 - {}^{9}P_2 + {}^{10}P_4 - {}^{9}P_3$

$= 9 + 90 - 9 + 720 - 72 + 5040 - 504$

$= 9 + 81 + 648 + 4536$

$= 5274$

OR

Total number of numbers formed with the digits 2, 3, 4, 5 taken all at a time $= {}^{4}P_4 = 4! = 24$

In order to find the sum of these 24 numbers, we shall find the sum of digits at unit's, ten's, hundred's and thousand's places of all these 24 numbers.

When the digits are at unit's place.

Each of 2, 3, 4, 5 occurs in unit's place in 3! = 6 times.

Total for such digits in the unit's place in all numbers = (2 + 3 + 4 + 5) × 3! = 84

Same is the case for the remaining places.

Hence, the sum of all the numbers = 84 (1 + 10 + 100 + 1000) = 93324

21. Let P(x, y) be any point on the locus.

By definition,

$|PS_1| = |PS_2| = 2$

$$\sqrt{(x-4)^2+(y-0)^2}-\sqrt{(x+4)^2+(y-0)^2}=2$$

$$\sqrt{(x-4)^2+(y-0)^2}=2+\sqrt{(x+4)^2+(y-0)^2}$$

$$(x-4)^2+(y-0)^2=4+4\sqrt{(x+4)^2+(y-0)^2}+(x+4)^2+y^2$$

$$x^2-8x+16+y^2=4+4\sqrt{(x+4)^2+y^2}+x^2+8x+16+y^2$$

$$4\sqrt{(x+4)^2+y^2}=-16x-4$$

$$\sqrt{(x+4)^2+y^2}=-4x-1$$

$$(x+4)^2+y^2=(-4x-1)^2$$

$$x^2+8x+16+y^2=16x^2+8x+1$$

$$15x^2-y^2=15$$

$$x^2-\frac{y^2}{15}=1$$

OR

$9x^2 - 16y^2 = 144$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$ ....(i)

$a^2 = 16, b^2 = 9$

a = 4 and b = 3

$b^2 = a^2(e^2 - 1)$

$9 = 16(e^2 - 1)$

$e^2 - 1 = \frac{9}{16}$

$e^2 = \frac{25}{16}$

$e = \frac{5}{4}$

Length of the transverse axis = 2a = 8

Length of the conjugate axis = 2b = 6

Co-ordinates of foci are (ae, 0) and (-ae, 0) i. e. (5, 0) and (-5, 0)

Vertices are (a, 0) and (-a, 0) i. e. (4, 0) and (-4, 0)

22. $y = \frac{1 - \sin x}{1 + \cos x}$

$$\frac{dy}{dx} = \frac{(1+\cos x)\frac{d}{dx}(1-\sin x) - (1-\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{(1+\cos x)(-\cos x) - (1-\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{-\cos x - \cos^2 x + \sin x - \sin^2 x}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\sin x - \cos x - 1}{(1+\cos x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2} - \cos\frac{\pi}{2} - 1}{\left(1+\cos\frac{\pi}{2}\right)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = \frac{1-0-1}{(1+0)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}}=0$$

OR

$$y=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+....$$

$$\frac{dy}{dx}=0+\frac{dx}{dx}+\frac{1}{2!}\frac{dx^2}{dx}+\frac{1}{3!}\frac{dx^3}{dx}+....$$

$$\frac{dy}{dx}=1+\frac{1}{2}\times 2x+\frac{1}{3\times 2}\times 3x^2+...$$

$$\frac{dy}{dx}=1+x+\frac{x^2}{2}+....$$

$$\frac{dy}{dx}=1+\frac{x}{1!}+\frac{x^2}{2!}+....$$

$$\frac{dy}{dx}=y$$

23. $4y^2 + 12x - 12y + 39 = 0$

$4y^2 - 12y = -12x - 39$

$4(y^2 - 3y) = -12x - 39$

$$4\left(y^2-3y+\frac{9}{4}\right)=-12x-39+9$$

$$4\left(y^2-\frac{3}{2}\right)^2=-12\left(x+\frac{5}{2}\right)$$

$$\left(y^2-\frac{3}{2}\right)^2=-3\left(x+\frac{5}{2}\right)......(i)$$

Shifting the origin to the point)-5/2, 3/2) without rotating the axes and denoting the new coordinates with respect to these axes by X and Y we get

$x = X + (-5/2)$, $y = Y + 3/2$

Using these relations (i) reduces to

$Y^2 = -3X$

This is the form of $Y^2 = -4aX$. On comparing a = ¾

The coordinates of the vertex with respect to new axes are (X = 0, Y = 0). So, coordinates of the vertex with respect to old axes are (-5/2, 3/2)

Focus : The coordinates of the focus of the parabola with respect to new axes are X = -3/4, Y = 0

So, coordinates of the focus with respect to old axes are (-13/4, 3/2)

Directrix: The equation of the directrix of the parabola with respect to new axes is X = ¾. So, equation of the directrix of the parabola with respect to old axes is x = -7/4

SECTION – D

24. $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

$= \sin 30^\circ (\sin 10^\circ \sin 50^\circ) \sin 70^\circ$

$$= \frac{1}{2}\sin 50^\circ \sin 10^\circ \sin 70^\circ$$

$$= \frac{1}{2}\times\frac{1}{2}(2\sin 50^\circ \sin 10^\circ)\sin 70^\circ$$

$$= \frac{1}{4}(2\sin 50^\circ \sin 10^\circ)\sin 70^\circ$$

$$= \frac{1}{4}\left[\cos(50^\circ - 10^\circ) - \cos(50^\circ + 10^\circ)\right]\sin 70^\circ$$ $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$$= \frac{1}{4}[\cos 40^\circ - \cos 60^\circ]\sin 70^\circ$$

$$= \frac{1}{4}[\sin 70^\circ \cos 40^\circ - \sin 70^\circ \cos 60^\circ]$$

$$= \frac{1}{4}\left[\sin 70^\circ \cos 40^\circ - \sin 70^\circ \frac{1}{2}\right]$$

$$= \frac{1}{8}[2\sin 70^\circ \cos 40^\circ - \sin 70^\circ]$$

$$= \frac{1}{8}\left[\sin(70^\circ + 40^\circ) + \sin(70^\circ - 40^\circ) - \sin 70^\circ\right]$$ $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$$= \frac{1}{8}[\sin 110^\circ + \sin 30^\circ - \sin 70^\circ]$$

$$= \frac{1}{8}\left[\sin(180^\circ - 70^\circ) + \sin 30^\circ - \sin 70^\circ\right]$$

$$= \frac{1}{8}[\sin 70^\circ + \sin 30^\circ - \sin 70^\circ]$$

$$= \frac{1}{8}\times\frac{1}{2}$$

$$= \frac{1}{16}$$

$$\therefore \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$$

OR

$\cos A \cos(60^\circ - A)\cos(60^\circ + A)$

$$= \frac{1}{2}\cos A\left[2\cos(60^\circ - A)\cos(60^\circ + A)\right]$$

$$= \frac{1}{2}\cos A\left[\cos(60^\circ - A + 60^\circ + A) + \cos(60^\circ - A - 60^\circ - A)\right]$$

$$= \frac{1}{2}\cos A\left[\cos 120° + \cos(-2A)\right]$$

$$= \frac{1}{2}\cos A\left[-\frac{1}{2} + \cos 2A\right]$$

$$= -\frac{1}{4}\cos A + \frac{1}{2}\cos A \cos 2A$$

$$= -\frac{1}{4}\cos A + \frac{1}{4} 2\cos A \cos 2A$$

$$= -\frac{1}{4}\cos A + \frac{1}{4}\left[\cos(2A + A) + \cos(2A - A)\right]$$

$$= -\frac{1}{\quad} \qquad \frac{1}{\quad} \qquad + \cos A]$$

$$= \frac{1}{4}\cos 3A$$

$$\therefore \cos A \cos(60° - A)\cos(60° + A) = \frac{1}{4}\cos 3A$$

25. We have n = 200, $\overline{X} = 40$, $\sigma = 15$

$$\overline{X} = \frac{1}{n}\sum x_i$$

$$\sum x_i = n\overline{X} = 200 \times 40 = 8000$$

Corrected $\sum x_i$ = Incorrect $\sum x_i$ – sum of incorrect values + Sum of correct values

= 8000 – 34 + 43 = 8009

Corrected mean = $\frac{8009}{200} = 40.045$

$\sigma = 15$

Variance = 15^2 = 225

$$\frac{1}{200}\sum x_i^2 - \left(\frac{1}{200}\sum x_i\right)^2 = 225$$

$$\frac{1}{200}\sum x_i^2 - \left(\frac{8000}{200}\right)^2$$

$$\frac{1}{200}\sum x_i^2 - 1600$$

$$\sum x_i^2 = 200 \times 1825 = 365000$$

Incorrect $\sum x_i^2 = 365000$

Corrected

$\sum x_i^2 = \text{incorrect} - \text{sum of squares of incorrect values} + \text{sum of squares of correct values}$

= 365000 – 34^2 + 43^2 = 365693

$$\text{Corrected } \sigma = \sqrt{\frac{1}{n}\text{corrected}\sum x_i^2 - \left(\frac{1}{n}\text{corrected}\sum x_i\right)^2} = \sqrt{\frac{365693}{200} - \left(\frac{8009}{200}\right)^2} = 14.995$$

26.

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$$\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \left(\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}\right)^3$$

$$\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}} = \left(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}\right)^3$$

$$\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2 = \left(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}\right)^{3\times 2}$$

$$\frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \left(\frac{1+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^3$$

$$\frac{1+\sin\theta}{1-\sin\theta} = \left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)^3$$

$$\frac{1+\sin\theta-(1-\sin\theta)}{1+\sin\theta+(1-\sin\theta)} = \frac{(1+\sin\alpha)^3-(1-\sin\alpha)^3}{(1+\sin\alpha)^3+(1-\sin\alpha)^3}$$

$$\frac{2\sin\theta}{2} = \frac{6\sin\alpha+2\sin^3\alpha}{2+6\sin^2\alpha}$$

$$\sin\theta = \frac{3\sin\alpha+\sin^3\alpha}{1+3\sin^2\alpha}$$

27. $x + 2y = 10$ or $x = 10 - 2y$

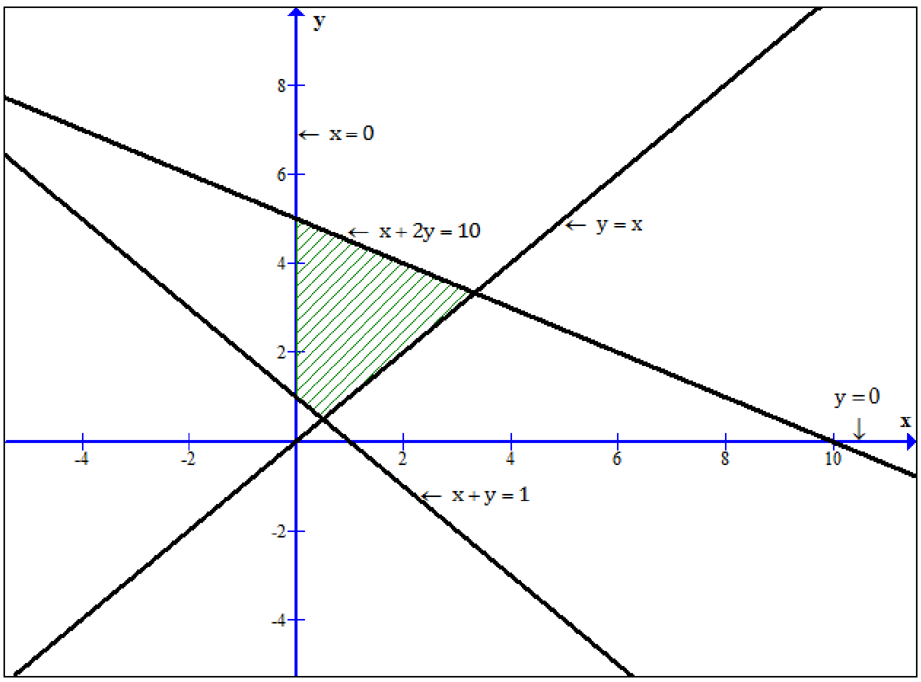
x	14	10	6
y	-2	0	2

$x + y = 1$ or $y = 1 - x$

x	−2	0	3
y	3	1	−2

$x - y = 0$ or $y = x$

x	−2	0	2
y	−2	0	2



OR

The amount of acid in 640 litres of the 8% solution= 8% of 640=$\frac{8\times640}{100}$

Let x litres of the 2% solution be added to obtain a solution between 4% and 6%

The amount of acid in x litres of the 2% solution $=\frac{2\times x}{100}$

The resultant amount $= 640 + x$

The amount of acid in $(640+x)$ litres solution is $=\frac{8\times640}{100}+\frac{2\times x}{100}$

Acid percentage of the solution now $=\dfrac{\frac{8\times640}{100}+\frac{2\times x}{100}}{640+x}\times100$

$$\Rightarrow 4<\dfrac{\frac{8\times640}{100}+\frac{2\times x}{100}}{640+x}\times100<6$$

$$\Rightarrow \frac{4(640+x)}{100}<\frac{8\times640}{100}+\frac{2\times x}{100}<\frac{6(640+x)}{100}$$

$$\Rightarrow 4(640+x)<5120+2x<6(640+x)$$

$$\Rightarrow 2(640+x)<2560+x<3(640+x)$$

$$\Rightarrow 2(640+x)<2560+x \text{ and } 2560+x<3(640+x)$$

$$\Rightarrow 1280+2x<2560+x \text{ and } 2560+x<1920+3x$$

$$\Rightarrow x<1280 \text{ and } 320<x$$

$$\Rightarrow 320<x<1280$$

Hence, the number of liters of 2% of acid which must be added should be more than 320 but less than 1280.

28. Let $P(n): S_n=\begin{cases}\frac{n(n+1)^2}{2}, & \text{when n is even}\\ \frac{n^2(n+1)}{2}, & \text{when n is odd}\end{cases}$

when $n=1, S_1=\frac{n^2(n+1)}{2}=\frac{1^2(1+1)}{2}=1$, which is true

Let the result holds for k i.e P(k) be true to prove P(k+1) to be true.

There are two cases

Case I: If k is odd then $(k+1)$ is even

To prove: $P(k+1): S_{k+1}=\frac{(k+1)((k+1)+1)^2}{2}=\frac{(k+1)(k+2)^2}{2}$

$P(k+1): P(k)+(k+1)^{th}$ term $=\frac{k^2(k+1)}{2}+2(k+1)^2$ (since $P(k)=S_k=\frac{k^2(k+1)}{2}$)

$$\frac{k^2(k+1)}{2}+2(k+1)^2=(k+1)\left[\frac{k^2}{2}+2(k+1)\right]=\frac{(k+1)}{2}\left[k^2+4(k+1)\right]$$

$$=\frac{(k+1)}{2}\left[k^2+4k+4\right]=\frac{(k+1)(k+2)^2}{2}$$

Sp P(k + 1) is true

Case II : k is even then (k+1) is odd

To prove : $P(k+1): S_{k+1}=\frac{(k+1)^2((k+1)+1)}{2}=\frac{(k+1)^2(k+2)}{2}$

$P(k+1): P(k)+(k+1)^{th}$ term $=\frac{k(k+1)^2}{2}+(k+1)^2$ (since, $P(k)=S_k=\frac{k(k+1)^2}{2}$)

$$=(k+1)^2\left[\frac{k}{2}+1\right]=\frac{(k+1)^2(k+2)}{2}$$

$\Rightarrow$ P(k + 1) is true

29. Interest to be paid with Installment 1 (S.I. on Rs. 6000 for 1 year) = $\frac{6000\times12\times1}{100}=720$

Interest to be paid with Installment 2 (S.I. on Rs. 5500 for 1 year) = $\frac{5500\times12\times1}{100}=660$

Interest to be paid with Installment 3 (S.I. on Rs. 5000 for 1 year) = $\frac{5000\times12\times1}{100}=600$

Interest to be paid with 12th Installment (S.I. On Rs 500 for 1 year) = $\frac{500\times12\times1}{100}=60$

Total interest paid = 720 + 660 + 600 + ... + 60

This forms an A.P., with a = 720 and d = −60

Sn = $\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow$ Sn = $\frac{12}{2}[2\times720+(12-1)(-60)]=4680$

The computer costed the student = 12000+ 4680= Rs. 16680

OR

$$\frac{1\times2^2+2\times3^2+3\times4^2+\text{....uptill the nth term}}{1^2\times2+2^2\times3+3^2\times4+\text{....uptill the nth term}}$$

Consider Numerator = $1\times2^2+2\times3^2+3\times4^2+\ldots+$ uptill the nth term

The nth term is $n(n+1)^2$

$\therefore 1\times2^2+2\times3^2+3\times4^2+$....uptill the nth term

$=\sum n(n+1)^2$

$=\sum n\ (n^2+1+2n)$

$=\sum\ (n^3+n+2n^2)=\sum n^3+2\sum n^2+\sum n$

$=\left[\frac{n(n+1)}{2}\right]^2+2\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$=\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+2\frac{(2n+1)}{3}+1\right]$

$=\frac{n(n+1)}{12}\left[3n^2+3n+8n+4+6\right]$

$=\frac{n(n+1)}{12}\left[3n^2+11n+10\right]$

$=\frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]$

Consider Denominator $=1^2\times2+2^2\times3+3^2\times4+$....uptill the nth term

The nth term is $n^2\ (n+1)$

$\therefore 1^2\times2+2^2\times3+3^2\times4+$....uptill the nth term

$=\sum n^2\ (n+1)$

$=\sum\ (n^3+n^2)=\sum n^3+\sum n^2$

$=\left[\frac{n(n+1)}{2}\right]^2+\frac{n(n+1)(2n+1)}{6}$

$=\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{3}\right]$

$=\frac{n(n+1)}{12}\left[3n^2+3n+4n+2\right]$

$=\frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]$

The given expression $=\dfrac{\frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]}{\frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]}=\frac{3n+5}{3n+1}$